Projections

Projection

DEF Let X be a set. The ***projection*** of the vector $\vec{v}$ onto X, written $\operatorname{proj}_X \vec{v}$, is the closest point in X to $\vec{v}$.

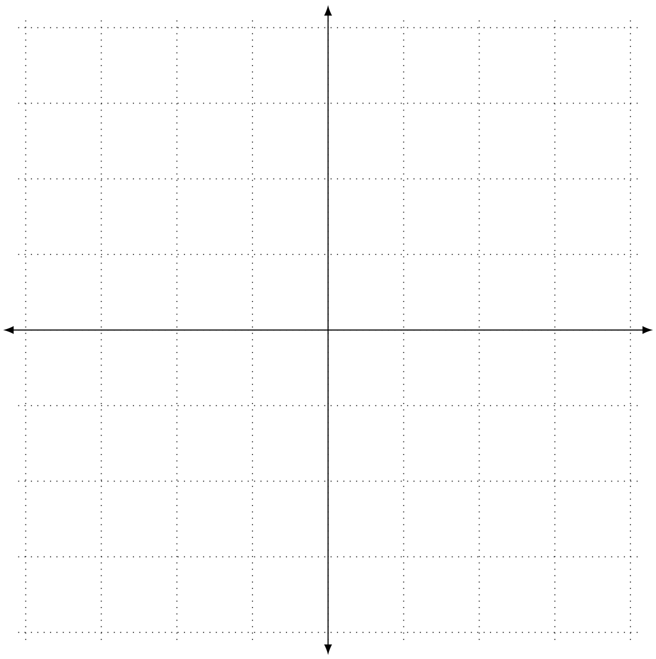
30 Let $\vec{a} = \begin{bmatrix}1\\0\end{bmatrix}$, $\vec{b} = \begin{bmatrix}4\\0\end{bmatrix}$, $\vec{v} = \begin{bmatrix}2\\2\end{bmatrix}$ and $\ell = \operatorname{span}\{\vec{a}\}$.

30.1 Draw $\vec{a}$, $\vec{b}$, and $\vec{v}$ in the same picture.

30.2 Find $\operatorname{proj}_{\{\vec{b}\}} \vec{v}$, $\operatorname{proj}_{\{\vec{a},\vec{b}\}} \vec{v}$.

30.3 Find $\operatorname{proj}_\ell \vec{v}$. (Recall that a quadratic $at^2 + bt + c$ has a minimum at $t = -\frac{b}{2a}$).

30.4 Is $\vec{v} - \operatorname{proj}_\ell \vec{v}$ a normal vector for ℓ? Why or why not?

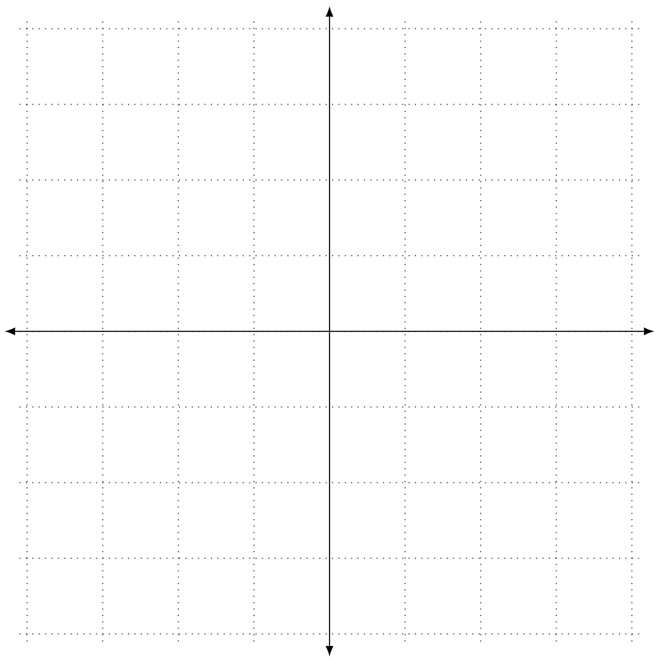


31 Let K be the line given in vector form by $\vec{x} = t\begin{bmatrix}1\\2\end{bmatrix} + \begin{bmatrix}1\\0\end{bmatrix}$ and let $\vec{c} = \begin{bmatrix}1\\3\end{bmatrix}$.

31.1 Make a sketch with $\vec{c}$, K, and $\text{proj}_K \vec{c}$ (you don't need to compute $\text{proj}_K \vec{c}$ exactly).

31.2 What should $(\vec{c} - \text{proj}_K \vec{c}) \cdot \begin{bmatrix}1\\2\end{bmatrix}$ be? Explain.

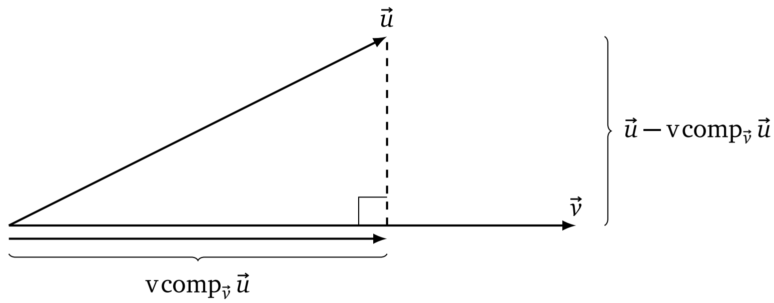
31.3 Use your formula from the previous part to find $\text{proj}_K \vec{c}$ *without* computing any distances.



Vector Components

DEFINITION

Let $\vec{u}$ and $\vec{v} \neq \vec{0}$ be vectors. The ***vector component of $\vec{u}$ in the $\vec{v}$ direction***, written $\operatorname{vcomp}_{\vec{v}} \vec{u}$, is the vector in the direction of $\vec{v}$ so that $\vec{u} - \operatorname{vcomp}_{\vec{v}} \vec{u}$ is orthogonal to $\vec{v}$.



32 Let $\vec{a}, \vec{b} \in \mathbb{R}^3$ be unknown vectors.

32.1 List two conditions that $\operatorname{vcomp}_{\vec{b}} \vec{a}$ must satisfy.

32.2 Find a formula for $\operatorname{vcomp}_{\vec{b}} \vec{a}$.